The 11^{th} Romanian Master of Mathematics Competition

Day 2: Saturday, February 23, 2019, Bucharest

Language: English

Problem 4. Prove that for every positive integer n there exists a (not necessarily convex) polygon with no three collinear vertices, which admits exactly n different triangulations.

(A *triangulation* is a dissection of the polygon into triangles by interior diagonals which have no common interior points with each other nor with the sides of the polygon.)

Problem 5. Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x + yf(x)) + f(xy) = f(x) + f(2019y),$$

for all real numbers x and y.

Problem 6. Find all pairs of integers (c, d), both greater than 1, such that the following holds:

For any monic polynomial Q of degree d with integer coefficients and for any prime $p > c(2c+1)$, there exists a set S of at most $\left(\frac{2c-1}{2c+1}\right)p$ integers, such that

$$\bigcup_{s \in S} \{s,\ Q(s),\ Q(Q(s)),\ Q(Q(Q(s))),\ \ldots\}$$

is a complete residue system modulo p (i.e., intersects with every residue class modulo p).

Each of the three problems is worth 7 points.
Time allowed $4\frac{1}{2}$ hours.