



# Romanian Master of Mathematics and Sciences

## 2011

### Physics Section

1. It is now the advent of the space age and cosmic flight is the norm. Convoys of spaceships crisscross the empty space between Earth and remote colonies. You are the commander of such a convoy, consisting of five rockets labeled from A to E in marching order. This is your very first mission but fortunately you don't have many responsibilities, since the entire cruise is set up to the last detail right from takeoff. There is even no need of a meter reading the cruising speed, because there are absolutely no nearby celestial bodies whatsoever to be taken as reference frame, and anyway the speed changes only at departure and arrival.

All you are provided with is a set of absolutely accurate and synchronized clocks, one for each rocket, and you also know the distance between the rockets, the same for each two consecutive ones. We will adopt this distance as length unit ("l.u.") and the units used in the readings of the clocks as time unit ("t.u."). (It is also important to stress the obvious fact that you cannot check any two of your clocks simultaneously.)

At some time during your journey you encounter a convoy absolutely identical in all respects (number of rockets, distance between them, magnitude of the velocity, and so on), returning to Earth from the same destination you are heading to. It is your duty to write down in the captain's log the exact time at which the other rockets (labeled from 1 to 5) pass by your ships. Here is the table documenting these events.

Time of passage	rocket 1	rocket 2	rocket 3	rocket 4	rocket 5
rocket A	00:00	03:00	06:00	09:00	12:00
rocket B	05:00	08:00	11:00	<b>14:00</b>	17:00
rocket C	<b>10:00</b>	13:00	16:00	19:00	22:00
rocket D	15:00	18:00	21:00	24:00	27:00
rocket E	20:00	23:00	26:00	29:00	32:00

The times written in red refer to another phenomenon recorded at the same moment as the passage of the incoming rockets.

By analyzing the above data, you are in a position to determine several things:

- The relative velocity  $v_{rel}$  of the other convoy with respect to yours.
- The velocity  $v_{abs}$  of both the convoys with respect to Earth.
- The relative velocity  $u_{rel}$  of the undetermined phenomenon with respect to you.
- The velocity  $u_{abs}$  of the undetermined phenomenon with respect to Earth.
- Determine the nature of the phenomenon and suggest an explanation for its presence.
- The table documenting these events as they are recorded in the captain's log of the other convoy, assuming that it starts also at time 00:00. Please use the table below as you answer sheet for this point. Please write in red the two times corresponding to the undetermined phenomenon in the first table.

Time of passage	rocket A	rocket B	rocket C	rocket D	rocket E
rocket 1	00:00				
rocket 2					
rocket 3					
rocket 4					
rocket 5					



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2. Unfortunately, there is a major problem with this manner of recording the data of a voyage: it only works within the proper reference frame, and it generally requires laborious transformations in order to be used by another observer. So now there is a very ingenious alternative way of charting a journey, devised at the dawn of the 21-st century by the American professor Nathaniel David Mermin. It is most simple, since it requires the use of only two triangles and a compass, and it allows all observers to work on the same document.

All space navigators need now to get familiar with this method and learn how to use it, especially new commanders such as you. It relies on only two graphical concepts. The first concept is the “equiloc”. It means events occurring in the same place at different moments of time. Any such events determine a straight line, as in the example below.

● (07:00)

● (05:00)

● (04:00)

The other concept is the “equitemp”. It means events occurring at the same time in different places. Any such events determine a straight line, as in the example below.

●  
B

●  
D

●  
E

For a given observer, all equilocs have the same slope, and all equitemps have the same slope, smaller than that of the equilocs. Also, please assume that the distances between points on an equiloc and the distances between points on an equitemp are correlated by the speed of light (e.g., two events 1 year apart on an equiloc are at the same distance as two events 1 light-year apart on an equitemp).

In what follows you are required to use your triangles and compass in order to draw events determining different equilocs and equitemps. In the end you will hopefully get a grasp on how to deal with these concepts in different reference frames using the same drawing.

Please look at the next page. It is your answer sheet for the points **g** to **j**, so please do not draw on it until you are quite sure of the answers. Use instead the scratch sheets having the same drawing, and come back to this one when you feel experienced enough.

**g.** You are given in red the two undetermined events in the initial table, and you are required to fill in the sketch with all the 23 other points, using your pencil.

**h.** As mentioned above, the diagram is able to serve all observers. Please point down on it the directions of the equilocs and of the equitemps for the returning convoy, and observe that its crew experiences exactly the same phenomena as yours.

**i.** Now, still on the same diagram, draw in green the directions of the equilocs and of the equitemps with respect to the reference frame of the Earth, and show that from that point of view both convoys look absolutely alike.

**j.** Suggest on the diagram a possible origin of the undetermined phenomenon. Also pin down in red the departure of your convoy from Earth and the arrival of the other one.



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At the bottom of this page you can see the Earth (E) and your Destination (D). The so called “Twin Paradox” asserts that if you make your trip to D and back to E, then when you will return the persons that you left back on Earth having the same age as you, will now be older than you and not the other way around.

**k.** By drawing your equilocs and equitemps and the Earth’s equilocs and equitemps, show that this is not at all a paradox, this is the pure reality. (Again, practice first on your scratch sheets and return to this one for your definitive answer.)



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**3.** Well, by now you most probably figured it out already what that unknown phenomenon was. The guys back on Earth sent you a message informing you that you were subject to the practical joke played on any new commander, a sort of “space baptism” tradition. They intentionally desynchronized your watches upon your departure, in order to lead you into believing that you are traveling at a speed close to that of the light, and consequently make you hope in a short and pleasant voyage (not to mention the Twin Paradox). In fact your speed is quite nonrelativistic, so you are in for a looong and booring trip!

**l.** In what manner and by how much did they alter the readings of your clocks? What is your real speed with respect to the reference frame of the Earth?

**m.** So in the end is this new method of describing relativistic phenomena correct or not? And is there anything inconsistent in this last part of the story (besides the fact that it is chronically lacking in talent)?