Language: English

Problem 4. Prove that there are infinitely many positive integers n such that $2^{2^n+1}+1$ is divisible by n but 2^n+1 is not.

Problem 5. Given a positive integer $n \geq 3$, colour each cell of an $n \times n$ square array with one of $\lfloor (n+2)^2/3 \rfloor$ colours, each colour being used at least once. Prove that there is some 1×3 or 3×1 rectangular subarray whose three cells are coloured with three different colours.

Problem 6. Let ABC be a triangle and let I and O denote its incentre and circumcentre respectively. Let ω_A be the circle through B and C which is tangent to the incircle of the triangle ABC; the circles ω_B and ω_C are defined similarly. The circles ω_B and ω_C meet at a point A' distinct from A; the points B' and C' are defined similarly. Prove that the lines AA', BB' and CC' are concurrent at a point on the line IO.

Each of the three problems is worth 7 points.
Time allowed $4\frac{1}{2}$ hours.