The 7th Romanian Master of Mathematics Competition

Day 2: Saturday, February 28, 2015, Bucharest

Language: English

Problem 4. Let ABC be a triangle, and let D be the point where the incircle meets side BC. Let J_b and J_c be the incentres of the triangles ABD and ACD, respectively. Prove that the circumcentre of the triangle AJ_bJ_c lies on the angle bisector of $\angle BAC$.

Problem 5. Let $p \geq 5$ be a prime number. For a positive integer k, let $R(k)$ be the remainder when k is divided by p, with $0 \leq R(k) \leq p-1$. Determine all positive integers $a < p$ such that, for every $m = 1, 2, \ldots, p-1$,

$$m + R(ma) > a.$$

Problem 6. Given a positive integer n, determine the largest real number μ satisfying the following condition: for every set C of $4n$ points in the interior of the unit square U, there exists a rectangle T contained in U such that

- the sides of T are parallel to the sides of U;
- the interior of T contains exactly one point of C;
- the area of T is at least μ.

Each of the three problems is worth 7 points.
Time allowed $4\frac{1}{2}$ hours.